Measurement of the Void Fraction in Two-Phase Flow by X-Ray Attenuation

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The use of an x-ray tube that emits a spectrum of energy was shown to be an effective source of monoenergetic radiation for the measurement of the void fraction in two-phase, steam-water flow with the proper selection of the tube wall thickness, x-ray tube operating conditions, collimation, and traverse time. Assuming monoenergetic radiation, equations were derived for the local and average void fraction and the statistical error associated with the measurement, and were confirmed to be applicable by using the mock-up technique. This consisted of simulating the two-phase, steam-water flow patterns of the experimental system with mock-ups of Lucite and air. Experimental data are presented to show that confidence could be placed in the measurements to within the probable statistical error, as all deviations of the measured values of the void fraction from the actual values were less than the probable error. Local and average void fractions for steam-water, two-phase flow are given. Application to other systems can be inferred from these results.

The description of the phase distribution in gas or vapor-liquid, two-phase flow has not been satisfactorily accomplished, and as one might expect, the subject has been the object of numerous studies. The measurement of the phase distribution, or more specifically the void fraction (the ratio of the cross-sectional area of the conduit occupied by the gas or vapor phase to the total cross-sectional area of the conduit), is tantamount to determining the average velocity of the individual phases. Therefore, various methods have been used to measure this variable in gas-liquid flow by many investigators (*1–4, 6–10, 12–16, 18, 19, 21–26, 28–32, 34–37*). These void fraction measurements of gas or vapor-liquid flow have been made in a wide variety of experimental systems: heated and unheated pipes, channels, nozzles, and simulated nuclear reactor sections. Air-water and steam-water systems were most commonly studied.

The purpose of this paper is to describe an effective and accurate method to measure the void fraction by radiation attenuation in two-phase, steam-water flow in a tube using an x-ray tube as a source of radiation. An x-ray tube has several advantages over radioisotopes. It is convenient, as it can be turned on and off at the user's will. It is generally a more economical source of radiation and can be obtained locally rather than from one of the suppliers of radioisotopes. Also, a special license is required by the Atomic Energy Commission to use radioisotopes. Further, and probably most important, the energy and intensity of the radiation beam can be adjusted over a wide range of energy and intensity, thus insuring optimum conditions. Of course, a radioisotope source is not as versatile.

This work will illustrate the use of an x-ray tube as a source of essentially monoenergetic radiation for the measurement of the void fraction. Previously it was thought that only particular radioisotopes, for example, thulium-170, that emit low energy, monoenergetic gamma rays could be used successfully. That is, low energy to obtain a significant attenuation by the two phase gas-liquid mixture, and monoenergetic to permit the use of equations that are solvable. To operate an x-ray tube for the emanation of radiation of a satisfactory energy to measure the void fraction, it was necessary that the potential difference between the target and filament be less than that required to obtain a sharp line spectrum. Therefore, the characteristic spectrum of energy was obtained. By using the mock-up technique for steam-water flow with the proper choice of conditions it was demonstrated experimentally that an x-ray tube may be considered a monoenergetic source of radiation. Moreover, the equations for the attenuation of monoenergetic radiation can be applied. As a result, data on the void fraction of two-phase, steam-water flow were obtained, and were invaluable in establishing the validity of a mathematical model to describe this flow (*29*).

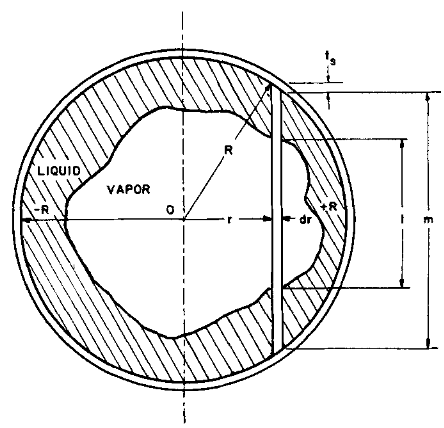

Fig. 1. Representation of tube cross section containing annular flow.

The possibility that an x-ray tube might be a suitable source of radiation was suggested by Marchaterre (*20*), who reported that a majority of the intensity of the gamma beam from the thulium-170 sources used in the Argonne National Laboratory work came not from the 0.08 mev. gamma beam emitted from the radioactive source, but from Bremsstrahlung radiation, a result of the 0.97 mev. beta particles impinging on the aluminum window of the source holder. Richardson (*30*) reported the energy spectrum for thulium-170, which showed the characteristic peaks rising from the spectrum obtained from the Bremsstrahlung radiation.

THEORETICAL

The following will be a development of the equations for the local and average void fractions for annular, gas-liquid, two-phase flow in a tube. Annular two-phase flow

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involves gas flowing in the core surrounded by liquid in the annulus. This flow type is depicted in Figure 1.

Derivation of Void Fraction Equation

The following relation describes the attenuation of a monoenergetic beam of gamma radiation passing through a thin homogenous absorbing medium of uniform thickness

$$I = I_o\, e^{-\sigma \rho t} \tag{1}$$

The following assumptions were made in order to obtain the relation for the local void fraction α, which is defined as the ratio of the chordal length of gas or vapor to the cordal length of the tube. First, a monoenergetic beam of gamma radiation is used which is described by Equation (1). Second, the beam of gamma radiation always passes through laminated layers. In Figure 1 a representation is given of the local void fraction in the cross section of a tube containing annular flow. Applying Equation (1) the attenuation of the beam through a chord of the tube when the tube is full of gas is given by

$$I_{MT} = I_o\, e^{-(\sigma_s \rho_s 2t_s + \sigma_g \rho_g m)} \tag{2}$$

The attenuation of the beam through the same chord of the tube when the tube is full of liquid is given by

$$I_F = I_o\, e^{-(\sigma_s \rho_s 2t_s + \sigma_l \rho_l m)} \tag{3}$$

The attenuation of the beam through the same chord of the tube with the tube containing two-phase flow, as depicted in Figure 1, is given by

$$I_{TP} = I_o\, e^{-[\sigma_s \rho_s 2t_s + \sigma_g \rho_g l + \sigma_l \rho_l (m-l)]} \tag{4}$$

The local void fraction is defined as the ratio of the chordal length of the gas to the chordal length of the tube; that is

$$\alpha = \frac{l}{m} \tag{5}$$

Therefore by combining Equations (2), (3), (4), and (5), the local void fraction can be expressed as

$$\alpha = \frac{\ln (I_{TP}/I_F)}{\ln (I_{MT}/I_F)} \tag{6}$$

It should be noted that the above expression (6) for the local void fraction is independent of the attenuation coefficients of a system of two phases.

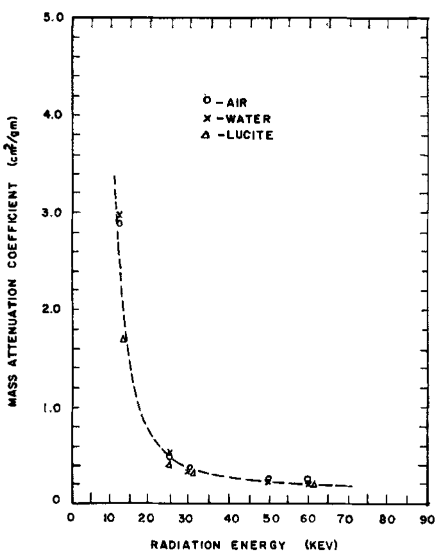

Fig. 2. Attenuation coefficients of air, water, and Lucite.

The average void fraction is the ratio of the cross-sectional area occupied by the steam to the cross-sectional area of the tube, and is given by

$$\alpha_{avg} = \frac{a_g}{A} = \frac{\int_{-R}^{R} l\,dr}{\pi R^2} \tag{7}$$

Equation (7) can be expressed in a more convenient form by using $l = \alpha m = 2\alpha\sqrt{R^2 - r^2}$ and $y = (r + R)/2R$ to shift the axis to the tube. The expression of α_{avg} becomes

$$\alpha_{avg} = \frac{8}{\pi}\int_0^1 \alpha\sqrt{y(1-y)}\,dy \tag{8}$$

Therefore, if the intensities I_{MT}, I_{TP}, and I_F are known as a function of y, the local void fraction α may be evaluated by Equation (6) as a function of y. Then the average void fraction can be calculated by Equation (8).

Error Analysis

The error introduced into the determination of the local and average void fractions from the measurement of the intensities will be considered, as this introduces the most significant error. However, there may be errors introduced into the void fraction determination due to characteristics of the particular two-phase flow pattern encountered if this method were to be applied to flows that oscillated, for example. This would have to be assessed for the particular system involved, however. For the annular flow pattern encountered here, oscillations did not occur that would affect the measurement.

To evaluate the error in the void fraction resulting from measuring the intensities, the fractional error in α can be expressed as (see reference *12*)

$$\frac{d\alpha}{\alpha} = \frac{\partial\alpha}{\partial I_{MT}}\cdot\frac{dI_{MT}}{\alpha} + \frac{\partial\alpha}{\partial I_F}\cdot\frac{dI_F}{\alpha} + \frac{\partial\alpha}{\partial I_{TP}}\frac{dI_{TP}}{\alpha} \tag{9}$$

Substituting positive values of the partial derivatives obtained from Equation (6) into Equation (9) gives the following equation for the maximum error in α.

$$e = \frac{1}{\ln\left(\frac{I_{MT}}{I_F}\right)\cdot\ln\left(\frac{I_{TP}}{I_F}\right)}\left[\frac{\Delta I_{TP}}{I_{TP}}\ln\left(\frac{I_{MT}}{I_F}\right) + \frac{\Delta I_{MT}}{I_{MT}}\ln\left(\frac{I_{TP}}{I_F}\right) + \frac{\Delta I_F}{I_F}\ln\left(\frac{I_M}{I_{TP}}\right)\right] \tag{10}$$

The error in the average void fraction is given by

$$e_{avg} = \frac{8}{\pi}\int_0^1 e\sqrt{y(1-y)}\,dy \tag{11}$$

Energy and Intensity Considerations

For two-phase, steam-water flow in a 1-in. pipe over a range of pressures from 14.7 to 100 lb./sq. in. abs., it will be shown that the radiation obtained from an x-ray tube operated at a peak voltage of 45 kv. will give a beam with essentially monoenergetic characteristics. By the use of a mock-up technique to simulate the steam-water flow this can be done by using material having essentially the same mass attenuation coefficient-radiation energy curve. The mock-up technique permitted the determination of the optimum x-ray tube operating conditions, proper collimation, traversing rate, and tube wall thickness before application to the real system. It is, of course, not necessary to go through the mock-up step if sufficiently accurate attenuation coefficient data and x-ray tube intensity data as a function of radiation energy are available to permit

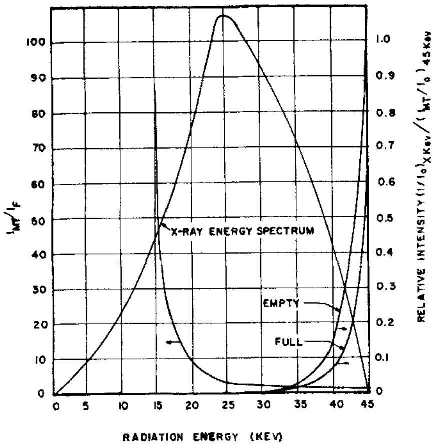

Fig. 3. Relative intensity vs. radiation energy.

calculations of these variables with reasonable confidence. Unfortunately, sufficiently accurate data are not generally available for a large number of systems, and this was the case for water. For a mock-up system, Lucite and air were used to simulate the steam-water system; this selection was based on the data for the mass attenuation coefficients reported by Grodstein (*11*) and Clark (*5*) as shown in Figure 2.

To show the effective, monoenergetic character of the radiation from an x-ray tube for the system, the following calculations were made. Using Figure 2 along with the data reported by Grodstein (*11*) for steel, a narrow beam of monoenergetic radiation was considered to pass through the diameter of the 1-in. O.D. tube of 0.035-in. wall thickness. The attenuation of this beam as a function of radiation energy was calculated for the tube full of water using the following equation.

$$\frac{I_F}{I_o} = e^{-(\sigma_s \rho_s 2t_s + \sigma_w \rho_w D)} \qquad (12)$$

The corresponding equation for the tube full of steam is

$$\frac{I_{MT}}{I_o} = e^{-(\sigma_s \rho_s 2t_s + \sigma_w \rho_G D)} \qquad (13)$$

The fraction change in intensity of the pipe full of water [Equation (12)] and the fractional change in the pipe full of steam [Equation (13)] were calculated as a function of radiation energy from 10 kev. to 45 kev. at 212°F. (14.7 lb./sq. in. abs.) and 328°F. (100 lb./sq. in. abs.). Also, the corresponding values were calculated for air and Lucite at 70°F. For a given radiation energy the fractional change in intensity for the tube full of water at 328°F. was only about 10% greater than the fractional change in intensity at 212°F. This was because the change in density of the water and of the steel was small over this range. The fractional change in intensity for the tube full of Lucite was about 10% less than the fractional change in intensity of the tube full of water at 212°F., as the density of Lucite (1.18 g./cc.) is nearly equal to that of water. Essentially equal fractional changes in intensity were obtained for calculations with the tube containing steam or air.

Also, the ratio (I_{MT}/I_F) was calculated for the above cases, as was the relative intensity.

$$\text{relative intensity} = \frac{(I/I_o)\ x\ \text{kev.}}{(I_{MT}/I_o)\ 45\ \text{kev.}} \qquad (14)$$

where $I = I_{MT}$ or I_F.

These results are plotted in Figure 3 for the case of the tube containing steam and water at 328°F. Corresponding plots for steam and water at 212°F. and Lucite and air at 70°F. were essentially the same, as discussed previously. Also shown in Figure 3 is the x-ray spectrum of a tungsten target operated at a peak voltage of 45 kev. (*32*).

It may be seen in Figure 3 that the relative intensity is essentially zero for 30 kev. and less. Thus, the tube walls attenuate essentially all of the beam below 30 kev. and only the photons of and energy between 45 and 30 kev. pass through the tube. If the x-ray spectrum is compared with the relative intensity, a large percentage of these photons have an energy between 33 and 42 kev. In this range of energy the attenuation coefficient is essentially constant (see Figure 2). This is illustrated by the curve for (I_{MT}/I_F), which is nearly horizontal. Therefore, based on these calculations an x-ray tube that has the characteristic spectrum of radiation energy may be considered as a monoenergetic radiation source for the conditions that were previously specified.

To satisfactorily determine the void fraction the beam must have sufficient intensity to permit a statistically significant measurement, and the difference in intensities between the tube full of vapor and that full of liquid must be statistically significant. For a particular tube this limits the amount of collimation. In this respect it is necessary that the beam collimated so that essentially parallel rays pass through the tube and a uniform beam of radiation is obtained. However, these considerations can be readily evaluated experimentally, as will be discussed subsequently.

EXPERIMENTAL SYSTEM

In Figure 4*a* an overall view of the void fraction apparatus is shown in position on the test section [see (*29*)] to measure the void fraction of steam-water flow. A Picker x-ray tube Model PX-3B with a tungsten target was mounted in a carriage

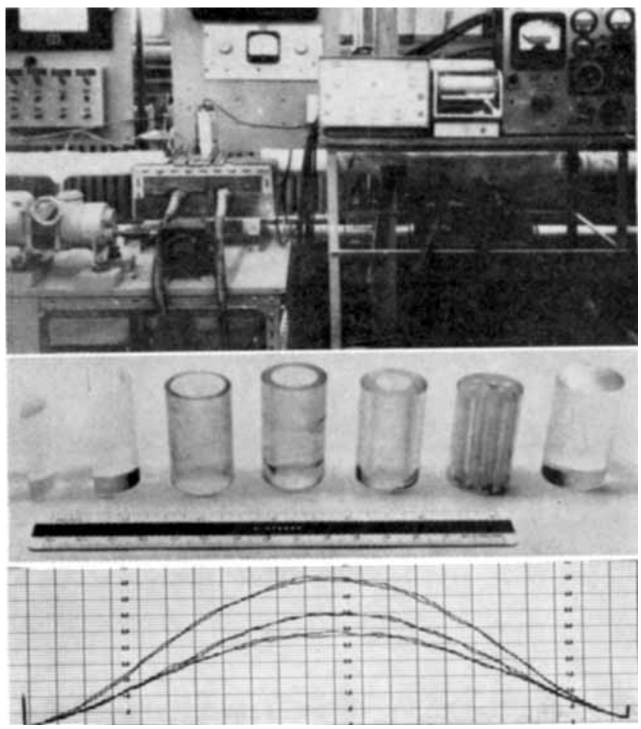

Fig. 4. Void fraction apparatus, Lucite mock-ups, and a typical composite traverse. (*a, top*) Overall view of the void fraction apparatus. (*b, middle*) Lucite mock-ups No. 1-7 from left to right. (*c, bottom*) Typical compositive traverse, mock-up No. 6.

and shielded with ⅛-in. lead sheet. The carriage was mounted on rails and moved in a horizontal straight line perpendicular to the test section by a lead screw. The lead screw was attached to a Boston Gear Reducer that was attached to a Master Speedranger variable speed gear reducer. This combination permitted varying the traverse time across the tube from 2 to 8 min. The x-ray tube, carriage, and speed reducers were securely mounted on a table with adjustable legs.

Collimation of the beam between the x-ray tube and the test section was accomplished by a 3/32-in. diameter hole drilled in a ¼-in thick lead sheet. Collimation between the top of the test section and the Nuclear Chicago Geiger-Mueller tube Type D-34 was accomplished by a 1/16-diam. hole drilled in a ½-in. thick sheet of lead. Alignment was accomplished by measuring the count rate, and both collimators were mounted securely to the carriage. Collimation below the test section provided a parallel beam of radiation from the x-ray tube and reduced the scatter of the radiation. Collimation of the beam above the test section allowed the Geiger-Mueller tube to receive only the essentially parallel beams of radiation that could pass up the 1/16-in. diam. hole. Any radiation that hit the lead surrounding the hole was essentially entirely attenuated.

The output from the Geiger-Mueller tube was measured by a Nuclear Chicago Analytical Count Rate Meter Model 1620A and recorded by a Texas Instrument Rectiriter-strip chart recorder. All intensity data were corrected for background and coincidence loss. The fractional statistical error in the count rate for this equipment was given by the following equation (27).

$$\frac{\Delta I}{I} = \frac{C}{\sqrt{2IC_t}} \quad (15)$$

where C is a constant depending on the confidence limit. The values of C and the corresponding confidence limits (17) are given in the following table.

TABLE 1. CONFIDENCE LIMITS CORRESPONDING TO C

C	Confidence limit	Reference name
0.6745	0.500	Probable error
1.65	0.900	Reliable error
2.58	0.990	99% error

The schematic wiring diagram for the x-ray tube, transformer, and control panel is given in reference 29. Controls were provided to vary the tube voltage and filament current. A G.E. 2.0 KVA Voltage Stabilizer was used to stabilize the line voltage to the control panel.

Mock-up Technique

A mock-up technique was used to establish experimentally that the void fraction could be accurately determined for a system that had essentially the same x-ray absorption characteristics (that is equal attenuation coefficients) as steam-water.

Also, for the mock-up system, it must be possible to accurately determine the void fraction by another means. As was previously discussed the Lucite-air system was chosen because it met these requirements. The plastic could be formed to simulate the liquid phase of two-phase flow patterns; the attenuation coefficients were essentially equal (as was seen); and the average void fraction was accurately determined by weighing the mock-ups and measuring their characteristic dimensions. Furthermore this technique helped establish the optimum conditions to perform the measurement. The conditions to be specified were x-ray tube voltage and filament current, count rate meter time constant, and traverse time across the tube for a specified tube wall thickness. It was desired to minimize the statistical error in counting, as this gave the best measure of the void fraction.

The Lucite mock-up is shown in Figure 4b. From left to right, the first two mock-ups simulated stratified flow; the second three annular flow; the sixth bubble flow with bubbles of 3/16-in. diameter; and the seventh all liquid flow. The actual void fractions of the mock-ups are given in Table 2. Although bubble flow is not a likely flow pattern for two-phase, steam-water flow under the conditions employed, this mock-up served to help evaluate the situation where the x-ray beam is of the same order of magnitude as the bubbles.

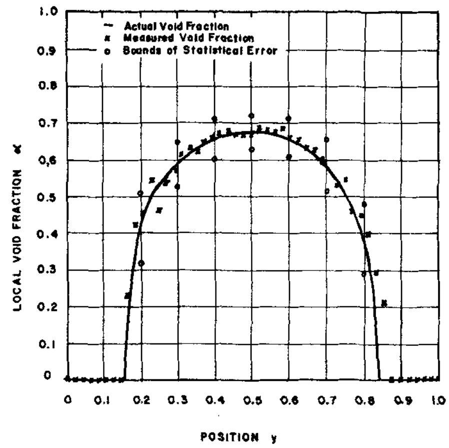

Fig. 5. Comparison of measured local void fraction and actual void fraction for mock-up No. 4.

The void fractions of the mock-ups were evaluated using a 1.5-in. long piece of the 0.035-in. wall thickness, 304 stainless steel tube used as the test section in the steam-water experiments. Traverses were made across the horizontal tube with the vertical beam of radiation in a plane perpendicular to the axis of the tube. For specified x-ray tube voltage and filament current, count rate meter time constant, and traverse time, at least two measurements of intensity versus time were recorded for both an empty tube as well as that containing the seven mock-ups. Composite curves for each set of data were prepared, and a typical example is shown in Figure 4c. Values for I_{MT}, I_F, and I_{TP} were read at forty-nine equally spaced intervals based on the tube I.D. from a curve smoothed through the composite curves. From these data α was computed as a function of y by Equation (6), and e by Equations (10) and (15). Also α_{avg} was computed by a Simpson's Rule integration of Equation (8), as was e_{avg} by Equation (11).

DISCUSSION OF LUCITE MOCK-UP RESULTS

The optimum operating conditions were experimentally determined for the peak voltage on the x-ray tube where the count rate was large enough to reduce the statistical error to an acceptable level. The best peak voltage was 45.0 kev. The optimum traverse time was experimentally determined so that at each point on the tube, as the tube was traversed, the measured count rate was within the statistical error of the actual count rate for the time constant used. The optimum traverse time was about 6 min. and optimum time constant was 10.0 sec. These optimum operating conditions gave sufficiently accurate data for the study. However, if a more accurate measurement were

TABLE 2. RESULTS OF VOID FRACTION MEASUREMENTS OF LUCITE MOCK-UPS AT A TRAVERSE TIME OF 5.7 MIN.

Mock-up No.	Void fraction: Actual value	Void fraction: Measured by x-ray attenuation	% deviation from actual	% statistical error
1	0.662	0.680	2.7	15.5
2	0.279	0.305	9.2	35.1
3	0.663	0.705	6.3	13.2
4	0.446	0.469	5.2	12.0
5	0.246	0.224	9.0	16.2
6	0.216	0.174	19.5	23.5
		Average	8.7	19.3

required, the statistical error could be reduced by increasing the traverse time and using a larger time constant on the count rate meter.

In Figure 5 a typical result of the traverse at these best operating conditions is shown. In Table 2 a comparison is given between the actual void fraction and the measured void fraction from the traverse at the optimum operating conditions. Also, a comparison is given in Table 2 between the deviation of the measured void fraction from the actual value and the statistical error in the measured value. The statistical error was computed with a value of 0.6745 for C (the probable error) in Equation (17) (see Table 1). Referring to Table 2, it may be seen that the deviations of the measured void fractions from the actual values were less than the statistical error based on the probable error for these six tests. This was the case of all thirty-six tests as shown in Table 3* for traverse times from 2.3 to 8.0 min. Thus, when the void fraction measurement was used on the test section with two-phase, steam-water flow, confidence could be placed in the measured void fraction to the extent that the error in the measurement would nearly always lie within the statistical error based on the probable error.

Also included in Table 3 are results obtained using the "one-shot" method. This method employed 1/16 wide slits whose length was equal to the tube diameter for collimation of the x-ray beam in place of the previously described collimation. With the one-shot method, the average void fraction was computed directly by using Equation (6) [See Richardson (*30*) for a clear discussion of the errors involved in using the one-shot method.]

The selection of 5.7 min. as the optimum traverse time was based on the data presented in Table 3. Referring to Table 3, it is seen that there was a significant improvement in the statistical error by increasing the traverse time from 4.5 to 5.7 min., that is from an average of 40% down to 19%. However, a further increase to 8.0 min. showed no further significant improvement. In fact, for the available equipment, a traverse time of the order of 15 min. would be required for a further improvement using the 50-sec. time constants of the count rate meter. The accuracy obtained at the 5.7-min. traverse time was sufficient for that required for the void fraction measurement when the accuracy of the other measurements that were made on the system were considered.

It can also be seen in Table 3 that the error in the measured value for mock-up No. 6 was consistently higher than that of the other mock-ups. Mock-up No. 6 simulated bubble flow that is not a typical horizontal two-phase flow pattern. But, more important, the diameter of the simulated bubbles (3/16 in.) was of the same order of magnitude of the x-ray beam (1/16 in.) rather than much larger, as was the case of the other mock-ups. In this bubble flow case the x-ray beam could not be considered to always pass through laminated layers vapor and liquid. Thus errors were introduced as a result of this fact. This mock-up then served as a check on the accuracy that could be expected when the beam diameter approached the diameter of the voids. Richardson (*30*) discussed these errors in detail also.

DISCUSSION OF STEAM-WATER RESULTS

In Tables 4* and 5* data are given on the average void fraction and the local void fraction for a series of tests on the adiabatic, evaporating flow of steam and water in horizontal pipe. Details of these runs are included in reference 29. Essentially, the data from these experiments could be separated into two groups. The data grouped in Table 4 were for the local and average void fractions when radial temperature gradients were small. In this case thermodynamic equilibrium at all stages of the expansion and constant temperature across any section normal to the flow was approached. For these data the quality could be computed with accuracy by the annular flow equations (29), and is reported in Table 4 also. The data in Table 5 are for runs where radial temperature gradients were significant. The quality, as predicted by the annular flow equation, for these tests was not reported since the equations are based on the assumption that no radial temperature gradients exist. In fact, the predicted quality would be greater than the experimental value due to these temperature gradients. Typical results of the local void fraction are given in Figure 6 for run 10213.

ANALYSIS OF FLOW PATTERNS BASED ON THE LOCAL VOID FRACTION DISTRIBUTION

An indication of two-phase flow patterns may be obtained from examining the plot of the local void fraction vs. reduced tube diameter. For example, consider the results shown in Figure 5 for an annular flow mock-up. For annular flow, the local void fraction is zero until the vapor core is reached. The vapor core is then represented by the characteristically shaped curve in Figure 5. For a stratified flow with the liquid filling slightly less than one-half the tube cross section, the local void fraction is equal to one until the liquid is reached; then the local void fraction decreases as the center of the tube is approached. For the case of homogeneous, or fog flow, where the liquid and vapor are intimately and uniformly mixed, the local void fraction would be constant. For the more physically probable case of annular flow with liquid droplets in the vapor core, the peak of the characteristic curve of Figure 5 would be flattened. In Figure 6 a plot is shown of the local void fraction for annular steam-water flow in a horizontal tube. Upon comparing Figures 5 and 6, the interpretation is that the steam-water flow is annular with droplets entrained, as previously discussed. This interpretation was visually confirmed (*29*).

SUMMARY

It has been shown that an x-ray tube, even though it is a source of radiation that emits a spectrum of energy, could be considered an effective source of monoenergetic radiation for the measurement of the void fraction with

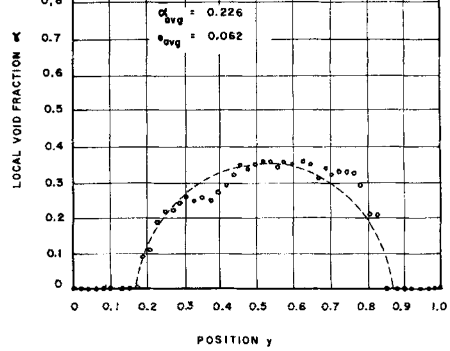

Fig. 6. Measured local void fraction for run 10213 of annular stream-water flow with entrained liquid drops.

* Tabular material has been deposited as document 8406 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or 35-mm. microfilm.

the proper selection of tube wall thickness, x-ray tube operating conditions, and collimation for the steam-water system. Equations derived for the local and average void fraction and the statistical error associated with the measurements based on monoenergetic radiation were shown applicable. It was said that the error in the steam-water void fractions was within the probable error, as the void fractions measured using a mock-up technique to simulate the flow were always less than the statistical probable error.

Although the discussion has been confined to the particular system of steam-water in 1-in. diameter pipe, the technique is applicable to other two-phase flow systems also. The equations presented here may be readily applied to other gas or vapor-liquid flows to establish the feasibility of the measurement for other systems.

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NOTATION

A = tube cross-sectional area, sq. ft.
a_g = cross-sectional area of vapor phase, sq. ft.
a_l = cross-sectional area of liquid phase, sq. ft.
C = constant from Table 1 for Equation (17), dimensionless
C_t = time constant, sec.
D = tube inside diameter, ft.
e = error in local void fraction, dimensionless
e_{avg} = error in average void fraction, dimensionless
g_c = conversion factor 32.174, ft.-lb.$_m$/sec.^{2}-lb.$_f$
I = intensity of gamma radiation, counts/min.
I_F = intensity of radiation passing through the tube full of liquid, counts/min.
I_{MT} = intensity of radiation passing through the tube full of gas, counts/min.
I_{TP} = intensity of radiation passing through the tube containing two-phase flow, counts/min.
I_o = intensity of radiation before passing through the tube, counts/min.
l = chord length of steam phase in two-phase flow, ft.
m = chord length of tube, ft.
R = tube radius, ft.
r = distance along the tube radius, ft.
t = thickness of homogenous absorbing medium, ft.
t_s = a thickness of the tube wall as indicated in Figure 1, ft.
y = reduced diameter, dimensionless

Greek Symbols

α = local void fraction, dimensionless
α_{avg} = average void fraction, dimensionless
δ = resolution time, μsec.
ρ = density, lb.$_m$/cu. ft.
ρ_g = vapor density, lb.$_m$/cu. ft.
ρ_l = liquid density, lb.$_m$/cu.ft.
σ = mass attenuation coefficient, sq. cm./g.
σ_{air} = mass attenuation coefficient for air, sq. cm. g.
σ_g = mass attenuation coefficient for gas, sq. cm./g.
σ_l = mass attenuation coefficient for liquid, sq. cm./g.
σ_s = mass attenuation coefficient for tube, materials, sq. cm./g.
σ_w = mass attenuation coefficient for water, sq. cm./g.
Δ = forward difference operator, dimensionless

Subscripts

air = air
F = full of liquid
f = fluid in tube
g = gas
l = liquid
M = measured
MT = full of gas
o = initial condition
s = tube material
T = actual or true
TP = two-phase
w = water

LITERATURE CITED

1. Anderson, G. H., and B. G. Mantzouranis, *Chem. Eng. Sci.*, **12**, 233 (1960).
2. Adorni, N., I. Casagrande, L. Cravarolo, A. Hassid, M. Silvestri, and S. Villani, *Nuclear Sci. Abstr.*, **18**, No. 1, 398 (1964).
3. Ball, W. P., D. B. Langmuir, and R. W. Wright, *Trans. Am. Nuclear Soc.* (December, 1958).
4. Berenson, P. J., and R. A. Stone, paper presented at A.I.Ch.E. 51st Natl. Meeting, San Juan, Puerto Rico (1963).
5. Clark, G. L., "Applied X-Rays," p. 164, McGraw-Hill, New York (1955).
6. Condon, R. A., and N. C. Sher, *Nuclear Sci. Eng.*, **14**, 327 (1962).
7. Cook, W. H., Ph.D thesis, Illinois Inst. Technol.; AEC Rept. No. ANL-5621 (1956).
8. Dengler, C. E., and J. N. Addoms, *Chem. Eng. Progr. Symposium Ser. No. 18*, **52**, 95 (1956).
9. Gill, L. E., G. F. Hewitt, T. W. Hitchon, and P. M. C. Lacey, *Chem. Eng. Sci.*, **18**, 525 (1963).
10. Glovier, G. W., and M. M. Omer, paper presented at Joint Chemical Eng. Congr., Cleveland, Ohio (May 7-10, 1961).
11. Grodstein, G. W., "X-ray Attenuation Coefficients from 10 Kev to 100 Mev," p. 21, Natl. Bureau of Standard Circular-583, U. S. Department of Commerce (1957).
12. Hooker, H. H., and G. F. Popper, "A Gamma-Ray Attenuation Method for Void Fraction Determination in Experimental Boiling Heat Transfer Test Facilities, Rept. No. ANL-5766, U. S. Atomic Energy Commission (November, 1958).
13. Isbin, H. S., et al., *A.I.Ch.E. J.*, **5**, 427 (1959).
14. ———, N. C. Sher, and K. C. Eddy, *ibid.*, **3**, 136 (1957).
15. Johnson III, G. E., M.S. thesis, Georgia Inst. Technol., Atlanta, Georgia (1961).
16. Krasiakova, L. U., *Zh. Tekhn. Fiz.*, **22**, No. 4, 656 (1952).
17. Kuyper, A. C., *J. Chem. Educ.*, **36**, 128 (1959).
18. Magiros, P. G., and A. E. Dukler, *A.I.Ch.E. J.*, **6**, 463 (1960).
19. Marchaterre, J. F., M.S. thesis, Michigan Coll. Mining and Technol.; AEC Rept. ANL-5522 (1956).
20. ———, personal communication.
21. Martin, G. E., and E. W. Grohse, In "Advances in X-ray Analysis," Vol. 4, Plenum Press, New York (1961).
22. Martinelli, R. C., M. K. Boelter, et al., *Trans. Am. Soc. Mech. Engrs.*, **66**, 139-151 (1944).
23. McManus, H. N., *NP-9322, OOR Project No. 2117, Contract DA-30-115-ORD-993*, Cornell Univ., Ithaca, New York (October, 1959).
24. Miropolsky, Z. L., and R. I. Shneyerova, *Intern. J. Heat Mass Transfer*, **5**, 723 (1962).
25. Muir, J. F., and R. Eickhorn, In "Proceedings of the 1963 Heat Transfer and Fluid Mechanic Institute," pp. 183-204, Stanford Univ. Press, Palo Alto, California (1963).
26. Neal, L. G., and S. G. Bankoff, *A.I.Ch.E. J.*, **9**, 490 (1963).
27. *Operation and Maintenance Manual: Model 1620 Analytical Count-Rate Meter*, Nuclear Chicago Corporation, Chicago (March, 1955).
28. Petrick, M., *Two-Phase Air-Water Flow Phenomena, Rept. No. ANL-5787*, U. S. Atomic Energy Commission (March, 1958).

29. Pike, R. W., Ph.D. thesis, Georgia Inst. Technol., Atlanta, Georgia (1962).
30. Richardson, B. L., *Some Problems in Horizontal Two-Phase Two-Component Flow, Rept. No. ANL-5949,* U. S. Atomic Energy Commission (December, 1958).
31. Rowhani, S. Z., *Nuclear Sci. Eng.,* 14, 414 (1962).
32. Schwartz, K., *Investigation of the Density Distribution, Water, and Steam Velocities as Well as the Pressure Loss in Vertical and Horizontal Upflow Boiler Tubes," Rept. No. AEC-tr-2314,* U. S. Atomic Energy Commission (1954).
33. Semat, Henry, "Introduction to Atomic and Nuclear Physics," p. 135, Rinehart, New York (1956).
34. Taylor, N. H., G. F. Hewitt, and P. M. C. Lacey, *Chem. Eng. Sci.,* **18,** 537 (1963).
35. Wallis, G. B., D. A. Steen, C. B. Edgar, Jr., and S. N. Brenner, *Nuclear Sci. Abstr.,* **17,** No. 16, 3,625 (1963).
36. Wicks III, M. and A. E. Dukler, *A.I.Ch.E. J.,* **6,** 463 (1960).
37. Zmola, P. C., and R. V. Bailey, *Trans. Am. Soc. Mech. Engrs.,* **78,** 881-892 (1956).

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The Condensation of Mixed Vapors

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Although condensation from saturated mixed vapors upon a cold surface has been frequently explored experimentally, current theories describing this phenomenon have not been adequately tested. This work describes the condensation process in which two condensable components are involved and how, consequently, the mass transfer process is involved in their partial fractionation.

A discussion of the Bedingfield correction (*3*) for the effect of molecular weight difference of the diffusing components upon the mass transfer coefficient leads to its necessary association with the Ackermann factor (*4, 5*) for the general mass transfer effect. Data from two sets of experiments using different apparatuses demonstrate the use of this net corrective factor and also demonstrate the general applicability of the Drew-Colburn mass transfer coefficient, for transfer processes in which more than one component transfers from one phase to another.

The fundamental description of condensation from turbulent mixtures of vapors was set forth by Colburn and Hougen (*6*) for a binary mixture of one condensable and one noncondensable vapor. The mass transfer aspects of this approach were further generalized by Colburn and Drew (*5*) to include two condensable vapors and with less quantitative merit to include more than two components. The Colburn-Drew mass transfer coefficient F, which resulted from the above analyses, was defined by the following equation:

$$\omega = F \ln \frac{z - y_v}{z - y_o} \tag{1}$$

This coefficient is the same as the Stewart coefficient (*4*), although the defining equation is given in a different form.

For an adequate description of the condensation process, the above mass transfer equation is coupled with the heat transfer equation that is usually embodied as some form of the familiar Nusselt equation. The utility of this latter equation has been demonstrated experimentally and is well described in standard heat transfer texts (*11*).

Mass transfer considerations in the condensate phase further complicate the required analysis. This problem has been investigated experimentally by Kent and Pigford (*10*), who measured the changing vapor phase concentration as condensation continuously occurred along a vertical tube condenser. From the data of these investigators it is probably true that in ordinary condensation configurations the vapor phase offers the larger mass transfer resistance. Although the method for describing the diffusional process in the turbulent vapor stream from which more than one component condenses had been suggested in the early engineering literature, onlv Kent and Pigford have made use of this description.

The experiments described in this paper attempt to demonstrate directly the condensation process in which more than one component enters the liquid condensate phase.

THEORY

Equation (1) is derived from the effect of mass transfer upon the concentration profile in the vapor in the vicinity of the condensate surface. The fluid velocity profile in this region is assumed to be identical to that for which no mass transfer occurs; turbulence exists in the main body of flowing vapor and the model visualized is essentially that described by film theory. Bird, Stewart, and Lightfoot compared the effects of mass transfer as described by the penetration model and boundary layer theory also (*4*).

Equation (1) as written defines the coefficient F, which is independent of mass transfer within the limitations of film theory. However, to offer a complete discussion of any mass transfer process, its comparison with the equivalent heat transfer process must be made. Bedingfield and Drew (*3*) pointed out that when the diffusing species are of different molecular weights the convective velocity in the substantial derivative of the Fourier-Poisson equa-